What should the social planner do for keeping off the procrastination? *

Toshiaki Kouno †

Graduate School of Economics, University of Tokyo
and
Economic Research Center, Fujitsu Research Institute

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Abstract

We often see people procrastinate what should be done as soon as possible. Procrastination is often occurred in collective decision making. The stakeholders in collective decision making tend to waste time while trying to avoid responsibility and shifting blame to others. What should the social planner do for preventing the procrastination? In this paper, we investigate the effect of setup of the deadline. We show that the setup of deadline is insignificant whether the deadline has the punishment or not.

1 Introduction

Many persons have the experience to procrastinate what should be done as soon as possible. Procrastination generates the loss and the loss depresses the efficiency of society. In 'lost decade' of Japan, many reforms in politics, economy and society were delayed.

Why did (or do) we procrastinate? Nowadays, some psychologists and behavioral economists find that human beings have the innate propensity to procrastinate even if they do not have the over-optimistic expectation. It is consid-

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†Corresponding author: Toshiaki Kouno, Economic Research Center, Fujitsu Research Institute, New Pier Takeshiba South Tower, 1-16-1, Kaigan, Minato-ku, Tokyo 105-0022, Japan. Tel: +81 3 54018392; Fax: +81 3 54018438; E-mail:kouno.toshiaki@jp.fujitsu.com

ered that the real identity of their propensity is hyperbolic discounting. [1] The persons with hyperbolic discounting plays down the gain and the loss in the future and plays up the gain and the loss in the present. Therefore, they prefer larger loss to smaller loss when the smaller loss comes sooner in time than the larger. Needless to say, this pattern of discounting is dynamically inconsistent.

But, most of decision-making is taken not by one person but by several persons. Are the stakeholders in the collective decision-making hyperbolic discounters? The previous researches show that the procrastination happens if the stakeholders are not hyperbolic discounting. It is reason why they tend to waste time trying to avoid responsibility and shifting blame to others.

Maynard Smith (1974) writes the situation as the game in the biological context. Two animals face each other across the dish. If they deter, they share the profit fifty-fifty but rack up a large cost in the battle. If one compromises and the other deters, deterring animal gets all of the dish. If they compromise, they share the profit fifty-fifty amicably. We call the game of this type as war of attrition.

Alesina and Drazen(1991) applies the war of attrition to fiscal policy. [2] In the timing game, they assume that the game continues till any player compromises and that the value of dish is depreciated. In terms of game theory, corresponding payoff structure is known as the game of chicken. This game has pure strategy efficient equilibria and mixed strategy inefficient equilibrium. It is popular that mixed strategy inefficient equilibrium is only stable under best-response dynamics. In this game, it is social optimal that any player compromises at $t = 0$. However, the probability that the game does not end at $t = 0$ is positive in a stable equilibrium. Therefore, the expected time of stabilization is 'rational' delay.

Now, we implement the properties in the problem which can be written by the war of attrition:

(1)The existence of the problems itself produces the cost.

(2)The final disposal of the problems produces the cost.

(3)These costs increase as time goes on.

(4)Collective decision-making.

We can see these properties in many cases of joint ventures and politics. The cases include the nonperforming loans problem of Japan, global warming, and the grading in the omnibus lecture.

What should the social planner do for keeping off the procrastination? In this paper, we focus on the setup of the deadline. The setup of the deadline represses the procrastination itself. However, the setup changes the players' incentive. We show how the deadline and the punishment of breaking the deadline changes their incentives.

The remainder of this paper is organized as follows. Section 2 presents a benchmark model and equilibrium. Section 3 introduces the deadline. Section 4 concludes the paper. Formal proofs are collected in the Appendix.

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[1] Laibson(1997), O'Donoghue and Rabin(2006) and Benabou and Tirole(2002) give the explanation to this discounting.

[2] Fudenberg and Tirole(1991) investigated the theory of this game.

2 Benchmark

2.1 Model

At first, we present the model describing the reconciliation of interests. The existence of problems causes opportunity cost. We assume that the cost of the existence is $aD(t)$ at t and that the cost of final disposal is $bD(t)$. For simplicity, we assume $D(t)$ increases exponentially at the rate of $\gamma - 1(\geq 0)$. So, we set $D(t) = D_0\gamma^t(D_0 > 0)$.

We set discount factor δ. And we define $V(t)$ as the net present value from solving problems at t. So, this value is

$$\begin{aligned} V(t) &= -\delta^t bD(t) - a[D(0) + \delta D(1) + \cdots + \delta^{t-1}D(t-1)] \\ &= -(\gamma\delta)^t bD_0 - \frac{1-(\gamma\delta)^t}{1-\gamma\delta}aD_0. \end{aligned}$$

When should the final disposal be done? We set the period when the disposal should be done T^*. T^* is the solution of $\max_{t\geq 0} V(t)$.

i) $\gamma\delta \geq 1$ The presnet cost of final disposal and existence increases progressively. So, V(t) is decreasing function. Therefore, $T^* = 0$.

ii) $\gamma\delta < 1$ The present cost of final disposal decreases and the cost of existence increases. T^* is decided by the balance of the both sides.

$$\begin{aligned} &V(t+1) - V(t) \\ &\quad= -[(\gamma\delta)^{t+1} - (\gamma\delta)^t]bD_0 \\ &\qquad - [(1 + \cdots + (\gamma\delta)^{t-1} + (\gamma\delta)^t) - (1 + \cdots + (\gamma\delta)^{t-1})]aD_0 \\ &\quad= [b(1-\gamma\delta)(\gamma\delta)^t - a(\gamma\delta)^t]D_0. \end{aligned}$$

So, $V(t+1) - V(t) > 0 \Leftrightarrow (1-\gamma\delta)b > a$.

Therefore,

(a) if $1 - \gamma\delta > \frac{a}{b}$, $T^* = \infty$.

(b) if $1 - \gamma\delta < \frac{a}{b}$, $T^* = 0$.

We assume that the best timing of final disposal is 0 ($T^* = 0$). Hence,

Assumption .1 $1 - \gamma\delta < \frac{a}{b}$.

There are two players (A,B). The players' actions decide not only when the final disposal is done but also how much they share the cost of final disposal : $bD(t) = bD_0\gamma^t$. We assume that the actions which they can take are compromising(C) and deterring(D) at each period and that the game goes on until any player compromises.

We define the player i's action at t is $a_i(t)$ and the player i's share of the cost as $C(a_i(t), a_j(t))$. We assume that compromiseing player bears all the cost of final disposal if one player compromises and the other deters and that player A and player B split the cost if they compromise. So, $C(C,D) = bD(t), C(D,C) = 0$ and $C(C,C) = \frac{1}{2}bD(t)$.

If player A and player B deter, the final disposal is procrastinated. In this case, the existence of problems generates the opportunity cost, $aD(t)$. They split the cost between A and B. And they will negotiate in the next term again.

Now, we permit mixed strategy. So, we defined $p_i(t)$ as the probability that player i deters at t. $p_i(t) = 0$ means Compromise and $p_i(t) = 1$ means Deter. Hence, the strategy of player i is $(p_i(0), p_i(1), \cdots)$.

Under the situation that the final disposal has not done at t, we calculate the value estimated at 0 from eliminating bad loans at t. The player's payoff estimated at 0 is $F(t), L(t)$ or $B(t)$ if any player compromises at t.

When both players compromise, we define $B(t)$ as the payoff of each player. When player i compromises and player j $(i \neq j)$ deters, we define $L(t)$ as the payoff of player i and $F(t)$ as one of player j. These payoffs include the sunk cost at t. So, we can represent these payoff as follows:

$$\begin{aligned}
F(t) &= -\frac{1-(\gamma\delta)^t}{1-\gamma\delta}\frac{a}{2}D_0, \\
L(t) &= -\frac{1-(\gamma\delta)^t}{1-\gamma\delta}\frac{a}{2}D_0 - (\gamma\delta)^t bD_0 \\
&= F(t) - (\gamma\delta)^t bD_0, \\
B(t) &= -\frac{1-(\gamma\delta)^t}{1-\gamma\delta}\frac{a}{2}D_0 - \frac{1}{2}(\gamma\delta)^t bD_0 \\
&= F(t) - \frac{1}{2}(\gamma\delta)^t bD_0.
\end{aligned}$$

Now, we assume that the subgame is war of attrition. Hence,

Assumption .2 $L(n) > B(n+1) \Leftrightarrow 2 - \gamma\delta < \frac{a}{b}$.

If this assumption is violated, the subgames become prisoner's dilemma.

2.2 Equilibrium

Now, we investigate Subgame Perfect Nash Equilibrium in the infinite war of attrition. For the simplicity, we set the assumption as follows:

Assumption .3 *No player takes the strategy that he/she takes a stochastic action at any t and a nonstochastic action at the other t.*

At first, we investigate Pure Strategy Equilibrium. In the infinite war of attrition, the two stationary equilibrium are

(1) $p_A(t) = 1$ and $p_B(t) = 0$, $\forall t \geq 0$,

(2) $p_A(t) = 0$ and $p_B(t) = 1$, $\forall t \geq 0$.

In the former equilibrium, player B compromises at $t = 0$. So, player A does not bear at any cost. Even if player B does not compromise, player A deters infinitely. Therefore, the deposit is procrastinated and player B bear the loss, $\frac{a}{2}D_0 > 0$ at each period. For the minimization of these cost, player B compromises as soon as possible. And the game is finished at $t = 0$. If that helps, we can find some nonstationary equilibrium. In these equilibrium, either player compromises at $t = 0$ and the game is finished at $t = 0$.

At second, we investigate Mixed Strategy Equilibrium. There is a symmetric equilibrium where each player choices the same strategy. We write the mixed strategy in this symmetric equilibrium as $(\hat{p}(0), \hat{p}(1), \cdots, \hat{p}(t), \cdots)$. We can calculate $\hat{p}(t)$ used by the property that the profit of player i is not changed whether he/she compromises or deters in the mixed strategy equilibrium.

(a) When the player i compromises, If player j compromises, the player i's profit is $B(t) = F(t) - \frac{1}{2}(\gamma\delta)^t bD_0$.

If player j deters, the player i's profit is $L(t) = F(t) - (\gamma\delta)^t bD_0$.

Because the probability player j compromises is $1 - \hat{p}(t)$, The expected profit of player i is

$$\begin{aligned} E[V(t)|a_i(t) = C] &= [1 - \hat{p}(t)][F(t) - \frac{1}{2}(\gamma\delta)^t bD_0] + \hat{p}(t)[F(t) - (\gamma\delta)^t bD_0] \\ &= F(t) - \frac{1}{2}[1 + \hat{p}(t)](\gamma\delta)^t bD_0. \end{aligned}$$

(we write the action of player i at t as $a_i(t)$.)

(b) When the player i deters, If player j compromises, the player i's profit is $F(t)$.

If player j deters, the final deposit is procrastinated.

From the property of the mixed strategy equilibrium,

$$\begin{aligned} E[V(t+1)|a_i(t+1) = D] &= E[V(t+1)|a_i(t+1) = C] \\ &= F(t+1) - \frac{1}{2}[1 + \hat{p}(t+1)](\gamma\delta)^{t+1} bD_0. \end{aligned}$$

$$\begin{aligned} E[V(t)|a_i(t) = D] &= [1 - \hat{p}(t)]F(t) + \hat{p}(t)E[V(t+1)|a_i(t+1) = D] \\ &= [1 - \hat{p}(t)]F(t) + \hat{p}(t)[F(t+1) - \frac{1}{2}[1 + \hat{p}(t+1)](\gamma\delta)^{t+1} bD_0]. \end{aligned}$$

From (a) and (b),

$$F(t) - \frac{1}{2}[1 + \hat{p}(t)](\gamma\delta)^t bD_0 = [1 - \hat{p}(t)]F(t) + \hat{p}(t)[F(t+1) - \frac{1}{2}[1 + \hat{p}(t+1)](\gamma\delta)^{t+1} bD_0]$$

$$\Leftrightarrow$$

$$F(t)-\frac{1}{2}[1+\hat{p}(t)](\gamma\delta)^t bD_0=[1-\hat{p}(t)]F(t)+\hat{p}(t)[F(t)-(\gamma\delta)^t\frac{a}{2}D_0-\frac{1}{2}[1+\hat{p}(t+1)](\gamma\delta)^{t+1}bD_0]$$
$$\Leftrightarrow$$
$$-\frac{1}{2}[1+\hat{p}(t)](\gamma\delta)^t bD_0=\hat{p}(t)[-(\gamma\delta)^t\frac{a}{2}D_0-\frac{1}{2}[1+\hat{p}(t+1)](\gamma\delta)^{t+1}bD_0]$$
$$\Leftrightarrow$$
$$-[1+\hat{p}(t)](\gamma\delta)^t=\hat{p}(t)[-(\gamma\delta)^t\frac{a}{b}-[1+\hat{p}(t+1)](\gamma\delta)^{t+1}]$$
$$\Leftrightarrow$$
$$-[1+\hat{p}(t)]=\hat{p}(t)[-\frac{a}{b}-[1+\hat{p}(t+1)](\gamma\delta)]$$
$$\Leftrightarrow$$
$$\gamma\delta\hat{p}(t)\hat{p}(t+1)-[1-\gamma\delta-\frac{a}{b}]\hat{p}(t)-1=0$$

This suggests the following proposition.

Proposition .1 *In the infinite war of attrition, there are stationary mixed strategy equilibrium and pure strategy equilibrium. That is to say, nonstationary mixed strategy equilibrium does not exist.*

Proof: see Appendix.

From Proposition.1, we can write a stationary mixed strategy equilibrium is $(p^*, p^*, \cdots)$. And p^* satisfied $\gamma\delta {p^*}^2-[1-\gamma\delta-\frac{a}{b}]p^*-1=0$. From assumption 2, the quadratic equation $\gamma\delta p^2-[1-\gamma\delta-\frac{a}{b}]p-1=0$ has the real solution from 0 to 1.

Therefore, $p^*=\frac{1-\gamma\delta-\frac{a}{b}+\sqrt{(1-\gamma\delta-\frac{a}{b})^2+4\gamma\delta}}{2\gamma\delta}$.

Needless to say, the socially best period to deposit is $t=0$. But the probability to deposit at $t=0$ is $1-(p^*)^2$. So, we do not always acheive the social optimality. And the expected period to deposit (EPD) is

$$(1-{p^*}^2)\times 0+{p^*}^2(1-{p^*}^2)\times 1+\cdots+{p^*}^{2t}(1-{p^*}^2)\times t+\cdots=\frac{{p^*}^2}{1-{p^*}^2}.$$

Now, the evolutionary stable strategy is not pure strategy but mixed strategy: $(p^*, p^*, \cdots)$. So, plausible equilibrium is mixed strategy equilibrium.

3 Setup of the Deadline

Is the policy to force the final disposal by the deadline decided by the financial authority effective? The government in Japan came out with the urgent economic package in April 2001. Many economists think that this package promoted the final disposal and recovered Japanese economy. Some of them believe that the setup of deadline is effective because the setup of deadline of the final disposal is a part of the package.

In this section, we investigate the effect of setup of the deadline.

3.1 Simple Deadline

At first, we define deadline is at n as that all player is forced to compromise at $n+1$ if neither player has compromised till n. That is to say, the payoff profile at $t=n+1$ is $(B(n+1), B(n+1))$ when each player has deterred till n.

We show that there are two pure strategy equilibria as follows:

(1) $p_A(t)=1$ and $p_B(t)=0$, $\forall t \geq 0$,

(2) $p_A(t)=0$ and $p_B(t)=1$, $\forall t \geq 0$.

We investigate mixed strategy equilibrium [3] . We focus on a symmetric equilibrium where each player choices the same strategy. Now, we write $\hat{p}_s(t,n)$ as the probability the player deters at t in the case that the deadline is n. As in the infinite war of attrition, we show that

$$\gamma\delta\hat{p}_s(t,n)\hat{p}_s(t+1,n) - [1-\gamma\delta-\frac{a}{b}]\hat{p}_s(t,n) - 1 = 0 \text{ if } t < n.$$

Next, we calcurate $\hat{p}_s(n,n)$.

(a) When the player i compromises, If player j compromises, the player i's profit is $B(n) = F(n) - \frac{1}{2}(\gamma\delta)^n bD_0$.

If player j deters, the player i's profit is $L(n) = F(n) - (\gamma\delta)^n bD_0$.

Because the probability player j compromises is $1-\hat{p}_s(n,n)$, the expected profit of player i is

$$\begin{aligned} E[V(n)|a_i(n)=C] &= [1-\hat{p}_s(n,n)][F(n)-\frac{1}{2}(\gamma\delta)^n bD_0] + \hat{p}_s(n,n)[F(n)-(\gamma\delta)^n bD_0] \\ &= F(n) - \frac{1}{2}[1+\hat{p}_s(n,n)](\gamma\delta)^n bD_0. \end{aligned}$$

(b) When the player i deters, If player j compromises, the player i's profit is $F(n)$.

If player j deters, the final deposit is taken by act at $t=n+1$.

So, the player i's profit is $B(n+1) = F(n+1) - \frac{1}{2}(\gamma\delta)^{n+1}bD_0$

Because the probability player j compromises is $1-\hat{p}_s(n,n)$,

$$E[V(n)|a_i(n)=D] = [1-\hat{p}_s(n,n)]F(n) + \hat{p}_s(n,n)[F(n+1) - \frac{1}{2}(\gamma\delta)^{n+1}bD_0].$$

From (a) and (b),

$$\begin{aligned} F(n) - \frac{1}{2}[1+\hat{p}_s(n,n)](\gamma\delta)^n bD_0 &= [1-\hat{p}_s(n,n)]F(n) + \hat{p}_s(n,n)[F(n+1) - \frac{1}{2}(\gamma\delta)^{n+1}bD_0] \\ &\Leftrightarrow \\ F(n) - \frac{1}{2}[1+\hat{p}_s(n,n)](\gamma\delta)^n bD_0 &= [1-\hat{p}_s(n,n)]F(n) + \hat{p}_s(n,n)[F(n) - (\gamma\delta)^n \frac{a}{2}D_0 - \frac{1}{2}(\gamma\delta)^{n+1}bD_0] \end{aligned}$$

[3] This equilibrium is evolutionary stable strategy equilibrium as in the case of infinite game.

$$
\begin{aligned}
&\Leftrightarrow \\
-\frac{1}{2}[1+\hat{p}_s(n,n)](\gamma\delta)^n bD_0 &= \hat{p}_s(n,n)[-(\gamma\delta)^n \frac{a}{2} D_0 - \frac{1}{2}(\gamma\delta)^{n+1} bD_0] \\
&\Leftrightarrow \\
-[1+\hat{p}_s(n,n)](\gamma\delta)^n b &= \hat{p}_s(n,n)[-(\gamma\delta)^n a - (\gamma\delta)^{n+1} b] \\
&\Leftrightarrow \\
-[1+\hat{p}_s(n,n)] &= \hat{p}_s(n,n)[-\frac{a}{b} - (\gamma\delta)] \\
&\Leftrightarrow \\
\hat{p}_s(n,n) &= \frac{-1}{1-\gamma\delta-\frac{a}{b}}.
\end{aligned}
$$

Because we solve the game with deadline by backward induction,

$$\hat{p}_s(t,n) = \hat{p}_s(t-1,n-1) = \cdots = \hat{p}_s(0,n-t).$$

For the convenience, we define x as $\gamma\delta$ and y as $-1+\gamma\delta+\frac{a}{b}$. We have that

$$x\hat{p}_s(t,n)\hat{p}_s(t+1,n) + y\hat{p}_s(t,n) - 1 = 0.$$

Then, we can represent $\hat{p}_s(t,n) = \frac{c_{n-t}}{c_{n-t+1}}$ as $c_k = \alpha^k X + \beta^k Y$. We can show that $\alpha = \frac{y+\sqrt{y^2+4x}}{2}$,$\beta = \frac{y-\sqrt{y^2+4x}}{2}$, $X = \frac{\alpha}{\alpha-\beta}$, $Y = \frac{-\beta}{\alpha-\beta}$. So,

$$\hat{p}_s(n-t,n) = \frac{\alpha^t X + \beta^t Y}{\alpha^{t+1}X + \beta^{t+1}Y} = \frac{\frac{1}{\alpha} + \frac{1}{\alpha}(\frac{\beta}{\alpha})^t \frac{Y}{X}}{1 + (\frac{\beta}{\alpha})^{t+1}\frac{Y}{X}} = \frac{1}{\alpha}\frac{1-(\frac{\beta}{\alpha})^{t+1}}{1-(\frac{\beta}{\alpha})^{t+2}} = p^* \frac{1-(\frac{\beta}{\alpha})^{t+1}}{1-(\frac{\beta}{\alpha})^{t+2}}.$$

This equation suggests the following results.

Proposition .2 *If k is odd, $\hat{p}_s(n-k,n) < p^*$. If k is even or 0, $\hat{p}_s(n-k,n) > p^*$.*

Proof: see Appendix.

Proposition .3 $\lim_{n\to\infty} \hat{p}_s(t,n) = p^*$ *for all $t < n$.*

Proof: see Appendix.

We can represent the expected period to deposit in the game with simple deadline whose terminal node is n as $EPD_s(n)$.

Proposition .4 $\lim_{n\to\infty} EPD_s(n) = \frac{{p^*}^2}{1-{p^*}^2}.$

Proof: see Appendix.

$\frac{{p^*}^2}{1-{p^*}^2}$ is the expected period to deposit (EPD) in the infinite game. So, we show that the case in the simple deadline is closed to the infinite case if n is close to the infinity by Proposition.3 and .4. This phenomenon may be consistent with the intuition which many persons have.

However, $\hat{p}_s(t,n)$ vibrates around p^*. Many persons may feel that setting the deadline enlarges the probability of compromising. This vibration shows that the intuition is wrong. We explain this vibration and conversion in the next subsection.

3.2 Why does the sequence of $\hat{p}_s(t,n)$ vibrate?

We feel that setting the deadline enlarges the probability of compromising intuitively. However, Proposition 2 shows that this intuition is wrong.

We try the intuitive explanation of this phenomenon.

We focus on the payoff matrixes.

	Compromise	Deter
Compromise	$F(t) - \frac{1}{2}bD_0(\gamma\delta)^t$ $F(t) - \frac{1}{2}bD_0(\gamma\delta)^t$	$F(t)$ $F(t) - bD_0(\gamma\delta)^t$
Deter	$F(t) - bD_0(\gamma\delta)^t$ $F(t)$	$F(t+1) - \frac{1}{2}bD_0(1+\hat{p}(t+1))(\gamma\delta)^{t+1}$ $F(t+1) - \frac{1}{2}bD_0(1+\hat{p}(t+1))(\gamma\delta)^{t+1}$

Table 1: the payoff matrix in the infinite war of attrition

	Compromise	Deter
Compromise	$F(n) - \frac{1}{2}bD_0(\gamma\delta)^n$ $F(n) - \frac{1}{2}bD_0(\gamma\delta)^n$	$F(n)$ $F(n) - bD_0(\gamma\delta)^n$
Deter	$F(n) - bD_0(\gamma\delta)^n$ $F(n)$	$F(n+1) - \frac{1}{2}bD_0(\gamma\delta)^{n+1}$ $F(n+1) - \frac{1}{2}bD_0(\gamma\delta)^{n+1}$

Table 2: the payoff matrix in the war of attrition with deadline ($t = n$)

Two matrixes have the property of chicken game. They are the same matrixes except (D,D). And the payoff of (D,D) at $t = n$ in the game with deadline is larger than one at steady state in the infinite game.

Because the expected payoff taking C must be the same as the payoff taking D, the probability of deterring at $t = n$ in the game with deadline: $\hat{p}_s(n,n)$ must become larger than the probability at steady state in the infinite game p^* (see Figure 1).

To put it another way, bacause compulsory disposal is done at $t = n+1$, each player has the incentive not to bear all of the disposal cost at $t = n$ but to shift blame the half of the disposal cost to each other.

	Compromise	Deter
Compromise	$F(t) - \frac{1}{2}bD_0(\gamma\delta)^t$ $F(t) - \frac{1}{2}bD_0(\gamma\delta)^t$	$F(t)$ $F(t) - bD_0(\gamma\delta)^t$
Deter	$F(t) - bD_0(\gamma\delta)^t$ $F(t)$	$F(t+1) - \frac{1}{2}bD_0(1+\hat{p}_s(t+1,n))(\gamma\delta)^{t+1}$ $F(t+1) - \frac{1}{2}bD_0(1+\hat{p}_s(t+1,n))(\gamma\delta)^{t+1}$

Table 3: the payoff matrix in the war of attrition with deadline ($t < n$)

Next, the payoff matrix at $t < n$ is Table 3. Table 1 and Table 3 are the same matrixes excepted the probability at (D,D). When $\hat{p}_s(t+1,n) > p^*$, the

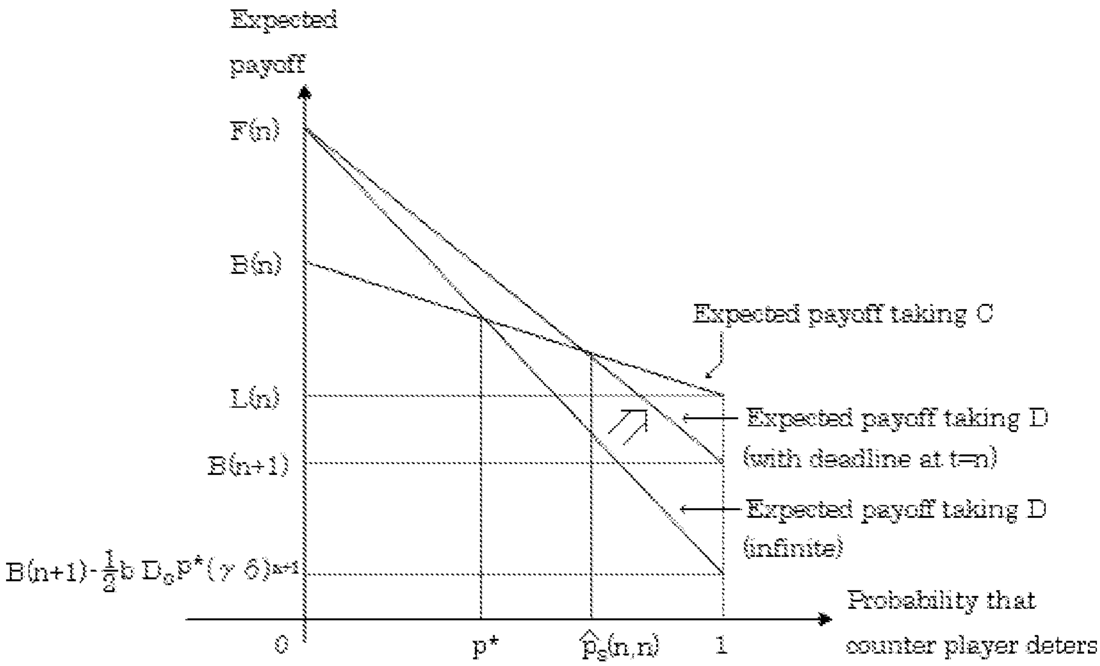

Figure 1: Expected payoff and probability that counter player deters

payoff in the game with deadline is lower than one at steady state in the infinite game if the player deters. Therefore, the probability of deterring $\hat{p}_s(t,n)$ must drop than p^*.

In fact, each player has more incentive to dispose at this period because the probability of deterring at the next period is up.

Vice versa, when $\hat{p}_s(t+1,n) < p^*$, the payoff in the game with deadline is higher than one at steady state in the infinite game if the player deters. Therefore, the probability of deterring $\hat{p}_s(t,n)$ must come up than p^* (see Figure 2).

In fact, each player has less incentive to dispose at this period because the probability of deterring at the next period is down.

This is the intuition of Proposition 2.

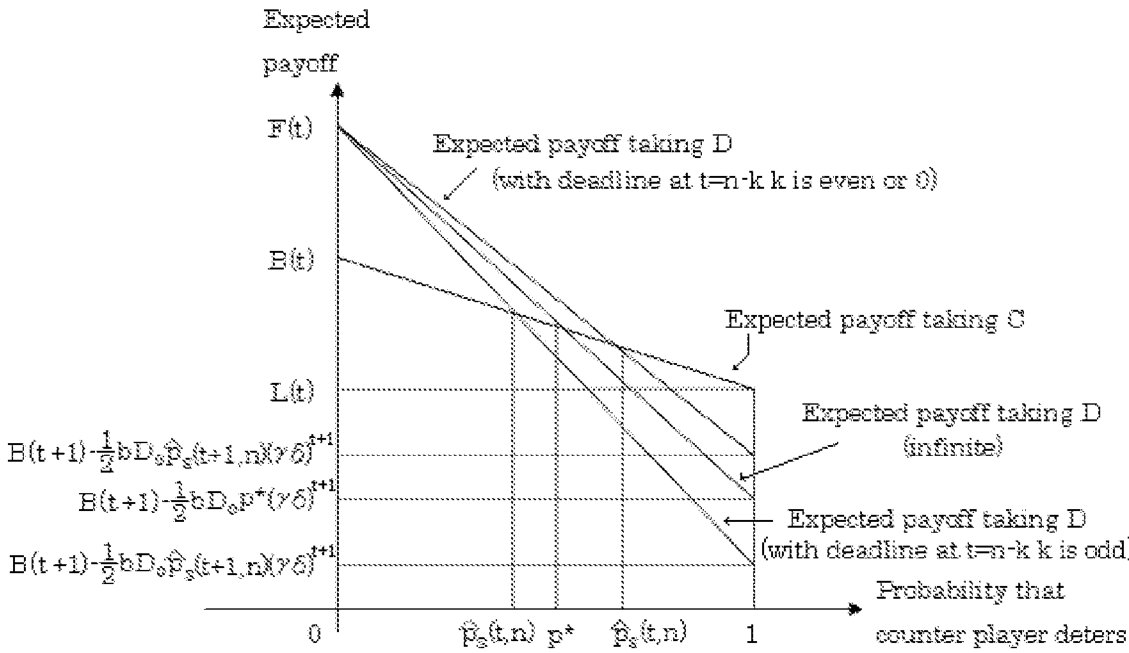

Figure 2: Expected payoff and probability that counter player deters

3.3 Is $EPD_s(n)$ larger or smaller than $\frac{p^{*2}}{1-p^{*2}}$?

It is hard to answer this question analytically because $EPD_s(n)$ is complex. (Please see the Proof of Proposition .4 in Appendix.)

Now, we try a simulation. We set $\delta = 0.2, \gamma = 3$ and $\frac{a}{b} = 1.41$. We have that $x = 0.6, y = 1.01$ and $p^* \approx 0.69946$. So,$\alpha \approx 1.43, \beta \approx -0.42, X \approx 0.77, Y \approx 0.23$.

We have that $\frac{p^{*2}}{1-p^{*2}} \approx 0.957879752$, $EPD_s(0) \approx 0.980296049, EPD_s(1) \approx 0.769642824, EPD_s(2) \approx 0.923807412, EPD_s(3) \approx 0.923255652, EPD_s(4) \approx 0.946256876, \cdots$.

We show this as Figure.3.

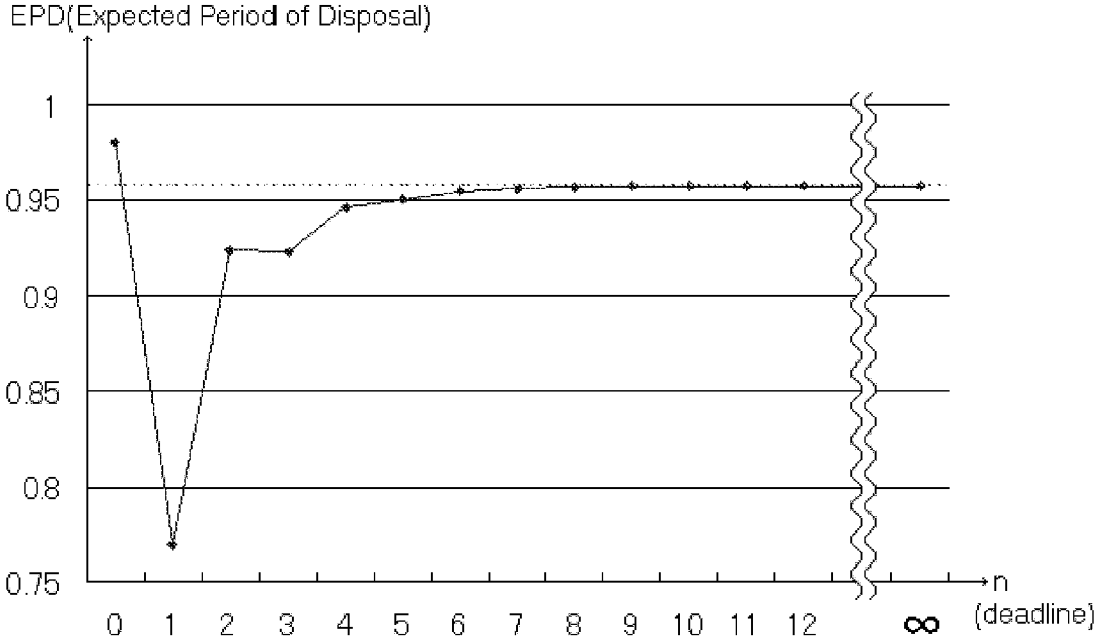

Figure 3: Expected period to deposit in simple deadline and infinite game

The above example shows that the regulation that the compulsory disposal will be done at the next period if the final disposal is not done at this period can be counterproductive. $\frac{p^{*2}}{1-p^{*2}} < EPD_s(0)$ means that the setup of the deadline encourage the procrastination.

The setting the deadline has two effects. The one is that the deadline prohibits the continued existence of the game. The other is that the deadline changes the incentive for the final disposal.

The latter effect at the odd period before the deadline is to encourage the procrastination. So, at the one period before the deadline, the extense of the deadline encourages the procrastination.

Now, we focus on the magnitude correlation between $EPD_s(0)$ and $\frac{p^{*2}}{1-p^{*2}}$.

$$EPD_s(0) > \frac{p^{*2}}{1-p^{*2}}$$
$$\Leftrightarrow$$

$$\frac{-y^2+2x+y\sqrt{y^2+4x}}{2}-1>0$$
$$\Leftrightarrow$$
$$\frac{-(-1+\gamma\delta+\frac{a}{b})^2+2\gamma\delta-1+\gamma\delta+\frac{a}{b}\sqrt{(-1+\gamma\delta+\frac{a}{b})^2+4\gamma\delta}}{2}-1>0 \qquad (1)$$

We show this area as Figure 4.

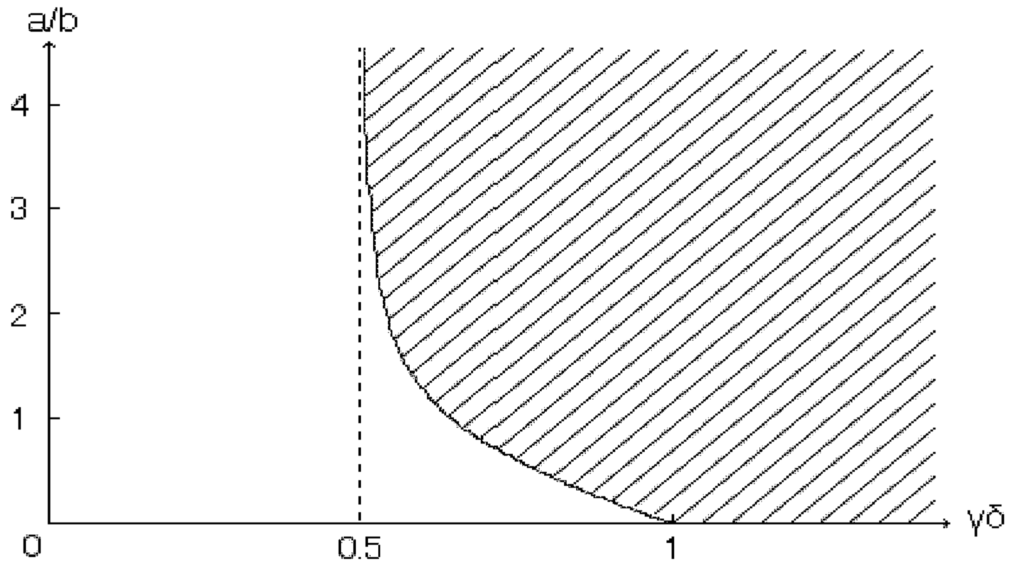

Figure 4: Expected payoff and probability that counter player deters

From Equation (1), When $\gamma\delta > 1$, the setting of the deadline always encourages the procrastination. However, the setting of the deadline always inhibits the procrastination when $\gamma\delta \leq 0.5$.

Now, we investigate the reason why the increases of $\gamma\delta$ has the propensity that the setting of the deadline encourages the procrastination. Plainly speaking, $\gamma\delta$ is the real growth rate of debt. The increase of the real growth rate of debt blocks the procrastination. But, the effect without deadline is more than one with deadline. Therefore, the increases of $\gamma\delta$ has the propensity that the setting of the deadline encourages the procrastination.

Now, we calcurate the case of $n = 1$.

$F(0) = 0$, $B(0) = -\frac{1}{2}bD_0$, $B(1) - \frac{1}{2}bD_0p^*\gamma\delta = -\frac{a}{2}D_0 - \frac{1}{2}bD_0\gamma\delta(1+p^*)$, $L(0) = -bD_0$.

If $\gamma\delta$ enlarges, p^* becomes small but $\hat{p}(0,0)$ does not change. So, the expected period to deposit at the infinite game becomes small. But the expected period to deposit at the finite game does not change. Therefore, the increases of $\gamma\delta$ has the propensity that the setting of the deadline encourages the procrastination.

3.4 Punitive deadline

In the above, we assume that the eliminating cost is split when the financial authority forces the final disposal. We show that the setup of deadline does not necessarily imply that the procrastination is prevented in this situation. But, it is ordinary that the authority will punish the bank if the bank has done nothing by the deadline which the authority set. So, we assume that the authority punish all the bank that have deterred by the deadline.

We set b as follows:

$$b = \begin{cases} b_0 & \text{if } t < n+1 \\ b_0 + b_p & \text{if } t = n+1 \end{cases} \quad (b_0 > 0, b_p > 0)$$

We consider b_p as the punishment from the authority. Needless to say, if $b_p = 0$, b is the same as the simple deadline.

Now, we replace Assumption .1 and .2. We set the assumption as follow.

Assumption .4 $2 - \gamma\delta < \frac{a}{b_0}$.

This assumption is the same as Assumption .1 and .2.

We write $\hat{p}_p(t,n)$ as the probability the player deters at t in the case that the punitive deadline is n. We show that

$$\gamma\delta\hat{p}_p(t,n)\hat{p}_p(t+1,n) - [1 - \gamma\delta - \tfrac{a}{b_0}]\hat{p}_p(t,n) - 1 = 0 \text{ if } t < n.$$

We calculate $\hat{p}_p(n,n)$ as follows by using the property that the expected payoff when the player compromises is the same as one when the player deters at n:

(a) When the player i compromises, If player j compromises, the player i's profit is $B(n) = F(n) - \frac{1}{2}(\gamma\delta)^n b_0 D_0$.

If player j deters, the player i's profit is $L(n) = F(n) - (\gamma\delta)^n b_0 D_0$.

Because the probability player j compromises is $1 - \hat{p}_p(n,n)$, the expected profit of player i is

$$\begin{aligned} E[V(n)|a_i(n) = C] &= [1 - \hat{p}_p(n,n)][F(n) - \frac{1}{2}(\gamma\delta)^n b_0 D_0] + \hat{p}_p(n,n)[F(n) - (\gamma\delta)^n b_0 D_0] \\ &= F(n) - \frac{1}{2}[1 + \hat{p}_p(n,n)](\gamma\delta)^n b_0 D_0. \end{aligned}$$

(b) When the player i deters, If player j compromises, the player i's profit is $F(n)$.

If player j deters, the final deposit is taken by act at $t = n+1$.

So, the player i's profit is $F(n+1) - \frac{1}{2}(\gamma\delta)^{n+1}(b_0 + b_p)D_0$

Because the probability player j compromises is $1 - \hat{p}_p(n,n)$,

$$\begin{aligned}
&E[V(n)|a_i(n) = D] \\
&\quad = [1 - \hat{p}_p(n,n)]F(n) + \hat{p}_p(n,n)[F(n+1) - \frac{1}{2}(\gamma\delta)^{n+1}(b_0 + b_p)D_0] \\
&\quad = [1 - \hat{p}_p(n,n)]F(n) + \hat{p}_p(n,n)[F(n) - (\gamma\delta)^n \frac{a}{2}D_0 - \frac{1}{2}(\gamma\delta)^{n+1}(b_0 + b_p)D_0] \\
&\quad = F(n) + \hat{p}_p(n,n)[-(\gamma\delta)^n \frac{a}{2}D_0 - \frac{1}{2}(\gamma\delta)^{n+1}(b_0 + b_p)D_0].
\end{aligned}$$

From (a) and (b),

$$\begin{aligned}
F(n) - \frac{1}{2}[1 + \hat{p}_p(n,n)](\gamma\delta)^n b_0 D_0 &= F(n) + \hat{p}_p(n,n)[-(\gamma\delta)^n \frac{a}{2}D_0 - \frac{1}{2}(\gamma\delta)^{n+1}(b_0 + b_p)D_0] \\
&\Leftrightarrow \\
-\frac{1}{2}[1 + \hat{p}_p(n,n)](\gamma\delta)^n b_0 D_0 &= \hat{p}_p(n,n)[-(\gamma\delta)^n \frac{a}{2}D_0 - \frac{1}{2}(\gamma\delta)^{n+1}(b_0 + b_p)D_0] \\
&\Leftrightarrow \\
-[1 + \hat{p}_p(n,n)](\gamma\delta)^n b_0 &= \hat{p}_p(n,n)[-(\gamma\delta)^n a - (\gamma\delta)^{n+1}(b_0 + b_p)] \\
&\Leftrightarrow \\
-[1 + \hat{p}_p(n,n)]b_0 &= \hat{p}_p(n,n)[-a - (\gamma\delta)(b_0 + b_p)] \\
&\Leftrightarrow \\
\hat{p}_p(n,n) &= \frac{-b_0}{b_0 - \gamma\delta(b_0 + b_p) - a}.
\end{aligned}$$

The recurrence formula in "punitive deadline" is the same as one in "simple deadline." However, $\hat{p}_s(n,n) \neq \hat{p}_p(n,n)$ if $b_p > 0$.

Then, we can represent $\hat{p}_p(t,n) = \frac{d_{n-t}}{d_{n-t+1}}$ as $d_k = \alpha^k Z + \beta^k W$. We can show that $Z = \frac{\alpha + x\frac{b_p}{b_0}}{\alpha - \beta}$, $W = \frac{\beta - x\frac{b_p}{b_0}}{\alpha - \beta}$. So,

$$\hat{p}_p(n-t,n) = \frac{\alpha^t Z + \beta^t W}{\alpha^{t+1} Z + \beta^{t+1} W} = \frac{\frac{1}{\alpha}Z + \frac{1}{\alpha}(\frac{\beta}{\alpha})^t W}{Z + (\frac{\beta}{\alpha})^{t+1} W} = \frac{1}{\alpha}\frac{Z + (\frac{\beta}{\alpha})^t W}{Z + (\frac{\beta}{\alpha})^{t+1} W} = p^* \frac{Z + (\frac{\beta}{\alpha})^t W}{Z + (\frac{\beta}{\alpha})^{t+1} W}.$$

This equation suggests the following propositions.

Proposition .5 *(a) When* $b_p > -\beta\frac{b_0}{x}$
If k *is odd,* $\hat{p}_p(n-k,n) > p^*$. *If* k *is even or 0,* $\hat{p}_p(n-k,n) < p^*$.
(b) When $b_p < -\beta\frac{b_0}{x}$
If k *is odd,* $\hat{p}_p(n-k,n) < p^*$. *If* k *is even or 0,* $\hat{p}_p(n-k,n) > p^*$.
(c) When $b_p = -\beta\frac{b_0}{x}$
$\hat{p}_p(n-k,n) = p^*$.

Proof: see Appendix.

Proposition .6 $\lim_{n\to\infty} \hat{p}_p(t,n) = p^*$ *for all* $t < n$.

Proof: see Appendix.

We can represent the expected period to deposit in the game with punitive deadline whose terminal node is n as $EPD_p(n)$.

Proposition .7 $\lim_{n\to\infty} EPD_p(n) = \dfrac{p^{*2}}{1-p^{*2}}$.

Proof: see Appendix.

Proposition .8 *If* k *is odd,* $\hat{p}_s(n-k+1,n) > \hat{p}_p(n-k,n)$. *If* k *is even,* $\hat{p}_s(n-k+1,n) < \hat{p}_p(n-k,n)$.

Proof: see Appendix.

Proposition .9

$$\lim_{b_p\to\infty} \hat{p}_p(t,n) = \begin{cases} \hat{p}_s(t,n+1) & \text{if } t \neq n \\ 0 & \text{if } t = n \end{cases}$$

Proof: see Appendix.

From Proposition .6 and .7, there is little point in punishment when n is large. From Proposition .5, the difference between the case in simple deadline and one in punitive deadline is little when b_p is small. These results may be consistent with your intuition.

But, the possibility of deterring does not become small uniformly if b_p becomes large. Even if b_p becomes larger than the threshold $(-\beta\frac{b_0}{x})$, the magnitude relation between $\hat{p}_p(n-k,n)$ and p^* becomes reversed only.

Each player has more incentive to dispose at $t=n$ because the player receives the large punishment if the game has not ended since $t=n$. The possibility of deterring is vibrated as the simple deadline because the recurrence formula in simple deadline is the same as one in punitive deadline. Therefore, the possibility of deterring does not become small uniformly.

From Proposition .9, however serious the punishment be, it move up the players' action only one period.

4 Concluding Remarks

In this paper, we investigate the policy of preventing the procrastination. We show that the setup of deadline makes the probability of deterring be up and down every one period even if the financial authority implements the punishment. And the punishment moves up the gameset at most one period. Therefore, we conclude that the setup of deadline is not the effective methods of keeping off the procrastination.

Finally, we must admit that our model is restrictive. For example, we assume that the game is symmetric. We think this assumption is moderate in nonperforming loan problems not in main-bank system. However, this assumption may not be moderate in labor strikes or main-bank system. The change of the act (or custom) may affect the result of this paper. [4] Extending our model to these directions remains for future research.

A Appendix

A.1 Proof of Proposition .1

We assume that non stationary mixed strategy equilibrium exists. This equilibrium can be represented by $(p(0), p(1), \cdots, p(t), \cdots)$ (for each t and s, $p(t) \neq p(s)$). For any $p(t)$, $\gamma\delta p(t)^2 - [1-\gamma\delta - a]p(t) - 1 = 0$.

Then, there is T which $p(T) > 1$. This is contradiction. So, non stationary mixed strategy equilibrium does not exist.

A.2 Proof of Proposition .2

Now, $\beta < 0$,$\alpha > 0$. $-y - \sqrt{y^2+4x} < -y + \sqrt{y^2+4x} \Rightarrow -\alpha < \beta < 0$.

Therefore, $-1 < \frac{\beta}{\alpha} < 0$.

a) k is odd

$$-(\frac{\beta}{\alpha})^{k+1} < 0 < -(\frac{\beta}{\alpha})^{k+2}$$

$$0 < 1 - (\frac{\beta}{\alpha})^{k+1} < 1 - (\frac{\beta}{\alpha})^{k+2}$$

$$\frac{1-(\frac{\beta}{\alpha})^{k+1}}{1-(\frac{\beta}{\alpha})^{k+2}} < 1$$

$$\hat{p}_s(n-k,n) = p^* \frac{1-(\frac{\beta}{\alpha})^{k+1}}{1-(\frac{\beta}{\alpha})^{k+2}} < p^*.$$

b) k is even

$$-(\frac{\beta}{\alpha})^{k+2} < 0 < -(\frac{\beta}{\alpha})^{k+1}$$

$$0 < 1 - (\frac{\beta}{\alpha})^{k+2} < 1 - (\frac{\beta}{\alpha})^{k+1}$$

$$\frac{1-(\frac{\beta}{\alpha})^{k+1}}{1-(\frac{\beta}{\alpha})^{k+2}} > 1$$

$$\hat{p}_s(n-k,n) = p^* \frac{1-(\frac{\beta}{\alpha})^{k+1}}{1-(\frac{\beta}{\alpha})^{k+2}} > p^*.$$

[4] Okuno-Fujiwara and Kouno(2007) shows that the role-contingent strategy can be used in the main-bank system and that this strategy makes pure strategy efficient equilibria stable.

A.3 Proof of Proposition .3

$$\lim_{n\to\infty} \hat{p}_s(t,n) = \lim_{n\to\infty} p^* \frac{1+(\frac{\beta}{\alpha})^{n-t+1}}{1+(\frac{\beta}{\alpha})^{n-t+2}} = p^*$$

A.4 Proof of Proposition .4

$$\begin{aligned}
EPD_s(n) &= (1-\hat{p}_s(0,n)^2)\times 0 \\
&+ \hat{p}_s(0,n)^2(1-\hat{p}_s(1,n)^2)\times 1 \\
&+ \hat{p}_s(0,n)^2\hat{p}_s(1,n)^2(1-\hat{p}_s(2,n)^2)\times 2 \\
&+ \cdots \\
&+ \hat{p}_s(0,n)^2\hat{p}_s(1,n)^2\cdots\hat{p}_s(n-1,n)^2(1-\hat{p}_s(n,n)^2)\times n \\
&+ \hat{p}_s(0,n)^2\hat{p}_s(1,n)^2\cdots\hat{p}_s(n-1,n)^2\hat{p}_s(n,n)^2\times(n+1) \\
&= (1-\frac{{c_n}^2}{{c_{n+1}}^2})\times 0 \\
&+ \frac{{c_n}^2}{{c_{n+1}}^2}(1-\frac{{c_{n-1}}^2}{{c_n}^2})\times 1 \\
&+ \frac{{c_n}^2}{{c_{n+1}}^2}\frac{{c_{n-1}}^2}{{c_n}^2}(1-\frac{{c_{n-2}}^2}{{c_{n-1}}^2})\times 2 \\
&+ \cdots \\
&+ \frac{{c_n}^2}{{c_{n+1}}^2}\frac{{c_{n-1}}^2}{{c_n}^2}\cdots\frac{{c_1}^2}{{c_2}^2}(1-\frac{{c_0}^2}{{c_1}^2})\times n \\
&+ \frac{{c_n}^2}{{c_{n+1}}^2}\frac{{c_{n-1}}^2}{{c_n}^2}\cdots\frac{{c_1}^2}{{c_2}^2}\frac{{c_0}^2}{{c_1}^2}\times(n+1) \\
&= \frac{{c_{n+1}}^2-{c_n}^2}{{c_{n+1}}^2}\times 0 \\
&+ \frac{{c_n}^2-{c_{n-1}}^2}{{c_{n+1}}^2}\times 1 \\
&+ \frac{{c_{n-1}}^2-{c_{n-2}}^2}{{c_{n+1}}^2}\times 2 \\
&+ \cdots \\
&+ \frac{{c_1}^2-{c_0}^2}{{c_{n+1}}^2}\times n \\
&+ \frac{{c_0}^2}{{c_{n+1}}^2}\times(n+1) \\
&= \frac{{c_n}^2+{c_{n-1}}^2+\cdots+{c_1}^2+{c_0}^2}{{c_{n+1}}^2}
\end{aligned}$$

Now,

$${c_n}^2+{c_{n-1}}^2+\cdots+{c_1}^2+{c_0}^2 = (X+Y)^2+(\alpha X+\beta Y)^2+\cdots+(\alpha^n X+\beta^n Y)^2$$

$$= \frac{1-\alpha^{2n+2}}{1-\alpha^2}X^2 + \frac{1-(\alpha\beta)^{n+1}}{1-\alpha\beta}2XY + \frac{1-\beta^{2n+2}}{1-\beta^2}Y^2$$

${c_{n+1}}^2 = \alpha^{2n+2}X^2 + 2(\alpha\beta)^{n+1}XY + \beta^{2n+2}Y^2$

So,

$$EPD_s(n) = \frac{\frac{1-\alpha^{2n+2}}{1-\alpha^2}X^2 + \frac{1-(\alpha\beta)^{n+1}}{1-\alpha\beta}2XY + \frac{1-\beta^{2n+2}}{1-\beta^2}Y^2}{\alpha^{2n+2}X^2 + 2(\alpha\beta)^{n+1}XY + \beta^{2n+2}Y^2}$$

$$= \frac{\frac{\frac{1}{\alpha^{2n+2}}-1}{1-\alpha^2}X^2 + \frac{\frac{1}{\alpha^{2n+2}}-(\frac{\beta}{\alpha})^{n+1}}{1-\alpha\beta}2XY + \frac{\frac{1}{\alpha^{2n+2}}-(\frac{\beta}{\alpha})^{2n+2}}{1-\beta^2}Y^2}{X^2 + 2(\frac{\beta}{\alpha})^{n+1}XY + (\frac{\beta}{\alpha})^{2n+2}Y^2}$$

$$\lim_{n\to\infty} EPD_s(n) = \frac{\frac{-1}{1-\alpha^2}X^2}{X^2} = \frac{-1}{1-\alpha^2} = \frac{{p^*}^2}{1-{p^*}^2}.$$

A.5 Proof of Proposition .5

At first, we must show $1 + (\frac{\beta}{\alpha})^k \frac{W}{Z} > 0$.

If $(\frac{\beta}{\alpha})^k \frac{W}{Z} \geq 0$, $1 + (\frac{\beta}{\alpha})^k \frac{W}{Z} \geq 1 > 0$.

Otherwise, (1)k is even and $\frac{W}{Z} < 0$ or (2)k is odd and $\frac{W}{Z} > 0$.

(1)k is even and $\frac{W}{Z} < 0$

$$1 + \frac{W}{Z} < 1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}$$

From $1 + \frac{W}{Z} = \frac{W+Z}{Z} = \frac{1}{Z} > 0$, $1 + (\frac{\beta}{\alpha})^k \frac{W}{Z} > 0$.

(2)k is odd and $\frac{W}{Z} > 0$

$$1 + \frac{\beta}{\alpha}\frac{W}{Z} \leq 1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}$$

From $1 + \frac{\beta}{\alpha}\frac{W}{Z} = \frac{\alpha Z + \beta W}{Z} = \frac{y + x\frac{b_p}{b_0}}{Z} > 0$, $1 + (\frac{\beta}{\alpha})^k \frac{W}{Z} > 0$.

So, we show $1 + (\frac{\beta}{\alpha})^k \frac{W}{Z} > 0$.

(a)$W < 0$ **(**$b_p > -\beta\frac{b_0}{x}$**) i)k is odd**

$$0 < 1 + (\frac{\beta}{\alpha})^{k+1}\frac{W}{Z} < 1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}$$

$$\frac{1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}}{1 + (\frac{\beta}{\alpha})^{k+1}\frac{W}{Z}} > 1$$

$$\hat{p}_p(n-k, n) = p^* \frac{1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}}{1 + (\frac{\beta}{\alpha})^{k+1}\frac{W}{Z}} > p^*$$

ii)k is even

$$0 < 1 + (\frac{\beta}{\alpha})^k \frac{W}{Z} < 1 + (\frac{\beta}{\alpha})^{k+1} \frac{W}{Z}$$
$$\frac{1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}}{1 + (\frac{\beta}{\alpha})^{k+1} \frac{W}{Z}} < 1$$
$$\hat{p}_p(n-k, n) = p^* \frac{1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}}{1 + (\frac{\beta}{\alpha})^{k+1} \frac{W}{Z}} < p^*$$

(b)$W > 0$ **(**$b_p < -\beta \frac{b_0}{x}$**) i)k is odd**

$$0 < 1 + (\frac{\beta}{\alpha})^k \frac{W}{Z} < 1 + (\frac{\beta}{\alpha})^{k+1} \frac{W}{Z}$$
$$\frac{1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}}{1 + (\frac{\beta}{\alpha})^{k+1} \frac{W}{Z}} < 1$$
$$\hat{p}_p(n-k, n) = p^* \frac{1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}}{1 + (\frac{\beta}{\alpha})^{k+1} \frac{W}{Z}} < p^*$$

ii)k is even

$$0 < 1 + (\frac{\beta}{\alpha})^{k+1} \frac{W}{Z} < 1 + (\frac{\beta}{\alpha})^{k} \frac{W}{Z}$$
$$\frac{1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}}{1 + (\frac{\beta}{\alpha})^{k+1} \frac{W}{Z}} > 1$$
$$\hat{p}_p(n-k, n) = p^* \frac{1 + (\frac{\beta}{\alpha})^k \frac{W}{Z}}{1 + (\frac{\beta}{\alpha})^{k+1} \frac{W}{Z}} > p^*$$

(c)W=0 ($b_p = -\beta \frac{b_0}{x}$**)**

$$\hat{p}_p(n-k.n) = p^* \frac{1 + (\frac{\beta}{\alpha})^k \times 0}{1 + (\frac{\beta}{\alpha})^{k+1} \times 0} = p^*$$

A.6 Proof of Proposition .6

$$\lim_{n\to\infty} \hat{p}_p(t, n) = \lim_{n\to\infty} p^* \frac{1 + (\frac{\beta}{\alpha})^{n-t} \frac{W}{Z}}{1 + (\frac{\beta}{\alpha})^{n-t+1} \frac{W}{Z}} = p^*$$

A.7 Proof of Proposition .7

$$\begin{aligned} EPD_p(n) &= (1 - \hat{p}_p(0, n)^2) \times 0 \\ &+ \hat{p}_p(0, n)^2 (1 - \hat{p}_p(1, n)^2) \times 1 \end{aligned}$$

$$
\begin{aligned}
&+ \quad \hat{p}_p(0,n)^2\hat{p}_p(1,n)^2(1-\hat{p}_p(2,n)^2) \times 2 \\
&+ \quad \cdots \\
&+ \quad \hat{p}_p(0,n)^2\hat{p}_p(1,n)^2\cdots\hat{p}_p(n-1,n)^2(1-\hat{p}_p(n,n)^2) \times n \\
&+ \quad \hat{p}_p(0,n)^2\hat{p}_p(1,n)^2\cdots\hat{p}_p(n-1,n)^2\hat{p}_p(n,n)^2 \times (n+1) \\
&= \quad (1-\frac{{d_n}^2}{{d_{n+1}}^2}) \times 0 \\
&+ \quad \frac{{d_n}^2}{{d_{n+1}}^2}(1-\frac{{d_{n-1}}^2}{{d_n}^2}) \times 1 \\
&+ \quad \frac{{d_n}^2}{{d_{n+1}}^2}\frac{{d_{n-1}}^2}{{d_n}^2}(1-\frac{{d_{n-2}}^2}{{d_{n-1}}^2}) \times 2 \\
&+ \quad \cdots \\
&+ \quad \frac{{d_n}^2}{{d_{n+1}}^2}\frac{{d_{n-1}}^2}{{d_n}^2}\cdots\frac{{d_1}^2}{{d_2}^2}(1-\frac{{d_0}^2}{{d_1}^2}) \times n \\
&+ \quad \frac{{d_n}^2}{{d_{n+1}}^2}\frac{{d_{n-1}}^2}{{d_n}^2}\cdots\frac{{d_1}^2}{{d_2}^2}\frac{{d_0}^2}{{d_1}^2} \times (n+1) \\
&= \quad \frac{{d_{n+1}}^2-{d_n}^2}{{d_{n+1}}^2} \times 0 \\
&+ \quad \frac{{d_n}^2-{d_{n-1}}^2}{{d_{n+1}}^2} \times 1 \\
&+ \quad \frac{{d_{n-1}}^2-{d_{n-2}}^2}{{d_{n+1}}^2} \times 2 \\
&+ \quad \cdots \\
&+ \quad \frac{{d_1}^2-{d_0}^2}{{d_{n+1}}^2} \times n \\
&+ \quad \frac{{d_0}^2}{{d_{n+1}}^2} \times (n+1) \\
&= \quad \frac{{d_n}^2+{d_{n-1}}^2+\cdots+{d_1}^2+{d_0}^2}{{d_{n+1}}^2}
\end{aligned}
$$

Now,

$$
\begin{aligned}
{d_n}^2+{d_{n-1}}^2+\cdots+{d_1}^2+{d_0}^2 &= (Z+W)^2+(\alpha Z+\beta W)^2+\cdots+(\alpha^n Z+\beta^n W)^2 \\
&= \frac{1-\alpha^{2n+2}}{1-\alpha^2}Z^2+\frac{1-(\alpha\beta)^{n+1}}{1-\alpha\beta}2ZW+\frac{1-\beta^{2n+2}}{1-\beta^2}W^2
\end{aligned}
$$

${d_{n+1}}^2 = \alpha^{2n+2}Z^2+2(\alpha\beta)^{n+1}ZW+\beta^{2n+2}W^2$
So,

$$
EPD_p(n) = \frac{\frac{1-\alpha^{2n+2}}{1-\alpha^2}Z^2+\frac{1-(\alpha\beta)^{n+1}}{1-\alpha\beta}2ZW+\frac{1-\beta^{2n+2}}{1-\beta^2}W^2}{\alpha^{2n+2}Z^2+2(\alpha\beta)^{n+1}ZW+\beta^{2n+2}W^2}
$$

$$= \frac{\frac{\frac{1}{\alpha^{2n+2}}-1}{1-\alpha^2}Z^2 + \frac{\frac{1}{\alpha^{2n+2}}-(\frac{\beta}{\alpha})^{n+1}}{1-\alpha\beta}2ZW + \frac{\frac{1}{\alpha^{2n+2}}-(\frac{\beta}{\alpha})^{2n+2}}{1-\beta^2}W^2}{Z^2+2(\frac{\beta}{\alpha})^{n+1}ZW+(\frac{\beta}{\alpha})^{2n+2}W^2}$$

$$\lim_{n\to\infty} EPD_p(n) = \frac{\frac{-1}{1-\alpha^2}Z^2}{Z^2} = \frac{-1}{1-\alpha^2} = \frac{{p^*}^2}{1-{p^*}^2}.$$

A.8 Proof of Proposition .8

$$\begin{aligned} -Z-W &= \frac{-\alpha - x\frac{b_p}{b_0} + \beta + x\frac{b_p}{b_0}}{\alpha-\beta} \\ &= \frac{-\alpha+\beta}{\alpha-\beta} = -1 < 0 \end{aligned}$$

From this, $Z > 0$ and $\frac{\beta}{\alpha} < 0 < 1$,

$$\begin{aligned} \frac{\beta}{\alpha}(-1-\frac{W}{Z}) &> -1-\frac{W}{Z} \\ -\frac{\beta}{\alpha}+\frac{W}{Z} &> -1+\frac{\beta}{\alpha}\frac{W}{Z}. \end{aligned}$$

a)k is odd

$$\begin{aligned} 1-(\frac{\beta}{\alpha})^{2k+1}\frac{W}{Z}-(\frac{\beta}{\alpha})^{k+1}+(\frac{\beta}{\alpha})^{k}\frac{W}{Z} &< 1-(\frac{\beta}{\alpha})^{2k+1}\frac{W}{Z}-(\frac{\beta}{\alpha})^{k}+(\frac{\beta}{\alpha})^{k+1}\frac{W}{Z} \\ (1+(\frac{\beta}{\alpha})^{k}\frac{W}{Z})(1-(\frac{\beta}{\alpha})^{k+1}) &< (1-(\frac{\beta}{\alpha})^{k})(1+(\frac{\beta}{\alpha})^{k+1}\frac{W}{Z}) \end{aligned}$$

From $1+(\frac{\beta}{\alpha})^{k+1}\frac{W}{Z} > 0$ and $1-(\frac{\beta}{\alpha})^{k+1} > 0$,

$$\frac{1+(\frac{\beta}{\alpha})^{k}\frac{W}{Z}}{1+(\frac{\beta}{\alpha})^{k+1}\frac{W}{Z}} < \frac{1-(\frac{\beta}{\alpha})^{k}}{1-(\frac{\beta}{\alpha})^{k+1}}.$$

Therefore, $\hat{p}_s(n-k+1,n) = p^*\frac{1-(\frac{\beta}{\alpha})^{k}}{1-(\frac{\beta}{\alpha})^{k+1}} > p^*\frac{1+(\frac{\beta}{\alpha})^{k}\frac{W}{Z}}{1+(\frac{\beta}{\alpha})^{k+1}\frac{W}{Z}} = \hat{p}_p(n-k,n)$

b)k is even

$$\begin{aligned} 1-(\frac{\beta}{\alpha})^{2k+1}\frac{W}{Z}-(\frac{\beta}{\alpha})^{k+1}+(\frac{\beta}{\alpha})^{k}\frac{W}{Z} &> 1-(\frac{\beta}{\alpha})^{2k+1}\frac{W}{Z}-(\frac{\beta}{\alpha})^{k}+(\frac{\beta}{\alpha})^{k+1}\frac{W}{Z} \\ (1+(\frac{\beta}{\alpha})^{k}\frac{W}{Z})(1-(\frac{\beta}{\alpha})^{k+1}) &> (1-(\frac{\beta}{\alpha})^{k})(1+(\frac{\beta}{\alpha})^{k+1}\frac{W}{Z}) \end{aligned}$$

From $1+(\frac{\beta}{\alpha})^{k+1}\frac{W}{Z} > 0$ and $1-(\frac{\beta}{\alpha})^{k+1} > 0$,

$$\frac{1+(\frac{\beta}{\alpha})^{k}\frac{W}{Z}}{1+(\frac{\beta}{\alpha})^{k+1}\frac{W}{Z}} > \frac{1-(\frac{\beta}{\alpha})^{k}}{1-(\frac{\beta}{\alpha})^{k+1}}.$$

Therefore, $\hat{p}_s(n-k+1,n) = p^*\frac{1-(\frac{\beta}{\alpha})^{k}}{1-(\frac{\beta}{\alpha})^{k+1}} < p^*\frac{1+(\frac{\beta}{\alpha})^{k}\frac{W}{Z}}{1+(\frac{\beta}{\alpha})^{k+1}\frac{W}{Z}} = \hat{p}_p(n-k,n)$

A.9 Proof of Proposition .9

$$\lim_{b_p\to\infty}\frac{W}{Z}=\lim_{b_p\to\infty}\frac{-\beta-x\frac{b_p}{b_0}}{\alpha+x\frac{b_p}{b_0}}=\lim_{b_p\to\infty}\frac{-\frac{\beta}{b_p}-\frac{x}{b_0}}{\frac{\alpha}{b_p}+\frac{x}{b_0}}=\frac{-\frac{x}{b_0}}{\frac{x}{b_0}}=-1$$

Therefore,

$$\lim_{b_p\to\infty}\hat{p}_p(t,n)=\lim_{b_p\to\infty}p^*\frac{1+(\frac{\beta}{\alpha})^{n-t}\frac{W}{Z}}{1+(\frac{\beta}{\alpha})^{n-t+1}\frac{W}{Z}}=p^*\frac{1-(\frac{\beta}{\alpha})^{n-t}}{1-(\frac{\beta}{\alpha})^{n-t+1}}$$

$$=\begin{cases}\hat{p}_s(t,n+1) & \text{if } t\neq n\\ 0 & \text{if } t=n\end{cases}$$

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