

# **Pay-As-You-Go Pension and the Incentives to Self-educate or to Educate Children\***

Economic Research Center, Fujitsu Research Institute

Toshiaki KOUNO<sup>†</sup>

April 23, 2009    Revised: May 26, 2010

## **Abstract**

We investigate the relationship between the pay-as-you-go (PAYG) pension scheme and the accumulation of human capital. We introduce the incentives for self-education into an overlapping generations model. The PAYG pension scheme encourages the incentives to educate children, but it discourages the incentives for their self-education. We show that the elasticity of substitution of human capital function affects whether the PAYG pension scheme hampers accumulating human capital or not when the incentives for self-education are considered. The argument implies that while studying the relationship between the pension scheme, education, and economic growth, we must focus on the decision maker with regard to education and on the function of the human capital.

**Keyword:** Pay-As-You-Go, pension, education, economic growth

**JEL classification number:** D90, H55, I20, O15

---

\* We are grateful to Dan Sasaki, Yasushi Iwamoto, Toshihiro Matsumura, and the participants of the seminar at the University of Tokyo and Asia University for their comments and suggestions. The views expressed in this paper are solely the author's own and do not necessarily reflect those of Fujitsu Research Institute.

† Corresponding author: Economic Research Center, Fujitsu Research Institute, New Pier Takeshiba South Tower, 1-16-1 Kaigan, Minato-ku, Tokyo 102-0022, Japan.  
Tel: +81-3-5401-8392. E-mail: [kouno.toshiaki@jp.fujitsu.com](mailto:kouno.toshiaki@jp.fujitsu.com)  
web: <http://home.e01.itscom.net/tkouno>

## **1. Introduction**

The public pension scheme has become a very important issue in many developed countries. Further, determining whether the public pension should be financed by the pay-as-you-go (PAYG) scheme or the funding scheme is a pressing question.

Feldstein (1974) shows that the PAYG pension scheme inhibits economic growth. He emphasizes that the PAYG scheme disturbs private wealth accumulation and aggregate capital accumulation. Blanchard (1990) indicates that the PAYG scheme is favored only under the condition that the population growth rate is higher than the marginal product of capital. Since developed countries have low birth rates, the PAYG scheme is not sustainable in such countries.

However, as Becker (1993) reveals, not only physical capital, but also human capital is a source of economic growth. Under the PAYG pension scheme, it is expected that people educate their children further (invest more human capital) because the accumulation of their children's human capital increases their pension benefit. Therefore, the PAYG scheme seems to encourage economic growth, as shown by Kemnitz and Wigger (2000) and by Docquier and Paddison (2003).

The idea that the PAYG pension scheme provides an incentive for investing in education has already been presented by Pogue and Sgontz (1977). Since then, many papers on this subject have been published.

The previous papers focus on the incentive for investing in educating children. However, human capital is accumulated not only by the education by the parent generation but also by self-education. In this paper, we investigate not only parents' incentive to

educate children, but also the children's incentive to self-educate in an overlapping generations model.

The assumption that the volume of education is determined by the parent generation might be suitable for primary and secondary education. However, not only the effort of parent generation, but also that of the children's generation plays an important role in on-the-job training and tertiary education (for example MBA program and Ph.D. program).

Previewing our conclusion, we show that the PAYG pension scheme encourages educating children because the more the human capital of children is, the more is the pension benefit for parents. On the other hand, we reveal that this scheme hampers self-education because the PAYG scheme is equivalent to taxing on children's labor. Furthermore, we show that the elasticity of substitution of the function of the human capital affects whether the latter effect offsets the former or not.

The rest of this paper is organized as follows. In Section 2, we present an overlapping generations model. In Section 3, we characterize the equilibrium. In Section 4, we consider comparative statics. Finally, in Section 5, we conclude the paper.

## **2. The Model**

We consider a three-period overlapping generations model. For simplicity, we assume that each generation consists of a representative individual. In the first period of generation  $t$ 's life ("youth"), he invests  $e_{t-1}$  in self-education. On youth, he takes out a loan to help pay for his self-education. We do not consider the liquidity constraint, for

the simplicity. Moreover, generation  $t-1$  educates generation  $t$  by investing  $g_{t-1}$ . The human capital of generation  $t$  (efficiency unit of labor),  $H_t$ , is assumed to be a function of  $e_{t-1}$  and  $g_{t-1}$ . We have for simplicity the following functional form.

$$H_t = ((1-u)e_{t-1}^\beta + ug_{t-1}^\beta)^{\alpha/\beta} \quad (0 < u < 1, \quad 0 < \alpha < \beta < 1).$$

In the middle age of generation  $t$ , the representative individual obtains wage income  $w_t H_t$ . He contributes to pension where the contribution rate is  $\theta_t$ . He also educates his child (generation  $t+1$ )  $g_t$ , consumes  $c_t^1$ , pays back  $(1+r_{t-1})e_{t-1}$ , and saves for old age  $s_t$ .

In his old age, generation  $t$  obtains pension benefit,  $\theta_{t+1}w_{t+1}H_{t+1}$ , and reclaims his savings, including interest,  $(1+r_t)s_t$ . By the assumption of a small open economy,  $r_t$  is exogenously given. Generation  $t$ 's consumption at  $t+1$  is  $c_{t+1}^2$ .

The utility function of generation  $t$  is given by

$$U_t = \log c_t^1 + \varepsilon \log c_{t+1}^2$$

From above, the constraints of generation  $t$  are

$$(1-\theta_t)w_t H_t = g_t + c_t^1 + s_t + (1+r_{t-1})e_{t-1}, \quad \theta_{t+1}w_{t+1}H_{t+1} + (1+r_t)s_t = c_{t+1}^2$$

$$\therefore c_t^1 + \frac{c_{t+1}^2}{1+r_t} = (1-\theta_t)w_t H_t + \frac{\theta_{t+1}w_{t+1}H_{t+1}}{1+r_t} - g_t - (1+r_{t-1})e_{t-1}.$$

We assume that generation  $t$  maximizes  $U_t$  subject to the constraints. In other words, we do not consider altruism in this paper.

### 3. Equilibrium

The optimality conditions with respect to  $e_{t-1}$ ,  $g_t$ ,  $c_t^1$ , and  $c_{t+1}^2$  are

$$(1-\theta_t)w_t \frac{\partial H_t}{\partial e_{t-1}} - (1+r_{t-1}) = 0, \text{ if } e_t > 0,$$

$$\frac{1}{c_t^1} = \frac{\varepsilon(1+r_t)}{c_{t+1}^2}, \text{ and}$$

$$-\frac{1}{c_t^1} + \frac{\varepsilon\theta_{t+1}w_{t+1}}{c_{t+1}^2} \frac{\partial H_{t+1}}{\partial g_t} = 0, \text{ if } g_t > 0 \text{ respectively.}$$

Rearranging the above equations, we have two equations:

$$\frac{\partial H_t}{\partial e_{t-1}} = \frac{\alpha}{\beta} ((1-u)e_{t-1}^\beta + ug_{t-1}^\beta)^{\alpha/\beta-1} \beta(1-u)e_{t-1}^{\beta-1} = \frac{1+r_{t-1}}{(1-\theta_t)w_t} \dots \textcircled{1}$$

$$\frac{\partial H_{t+1}}{\partial g_t} = \frac{\alpha}{\beta} ((1-u)e_t^\beta + ug_t^\beta)^{\alpha/\beta-1} \beta ug_t^{\beta-1} = \frac{1+r_t}{\theta_{t+1}w_{t+1}} \dots \textcircled{2}$$

We assume that the contribution rate, wage rate, and interest rate remain unchanged over time ( $\theta = \theta_0 = \theta_1 \dots$ ,  $w = w_0 = w_1 \dots$ ,  $r = r_0 = r_1 \dots$ ), focusing on the steady state. On the steady state, we can denote  $e = e_0 = \dots = e_{t-1} = e_t$  and  $g = g_0 = \dots = g_{t-1} = g_t$ .

From  $\textcircled{1}$  and  $\textcircled{2}$ ,  $\left(\frac{g}{e}\right)^{\beta-1} = \frac{1-\theta}{\theta} \frac{1-u}{u}$ . Therefore, we can denote

$$e = \theta^{\frac{1}{\beta-1}} u^{\frac{1}{\beta-1}} \cdot f(\theta), \text{ and}$$

$$g = (1-\theta)^{\frac{1}{\beta-1}} (1-u)^{\frac{1}{\beta-1}} \cdot f(\theta)$$

$$\text{where } f(\theta) \equiv \left[ \left\{ (1-u)\theta^{\frac{\beta}{\beta-1}} u^{\frac{\beta}{\beta-1}} + u(1-\theta)^{\frac{\beta}{\beta-1}} (1-u)^{\frac{\beta}{\beta-1}} \right\}^{\frac{\alpha}{\beta-1}} \theta(1-\theta)u(1-u) \frac{\alpha w}{1+r} \right]^{\frac{1}{1-\alpha}} .$$

These equations suggest the following proposition.

### **Proposition 1**

*As the contribution rate increases, the self-education decreases and the education for children increases.*

Proof: See Appendix.

Proposition 1 shows that PAYG pension scheme enhances the education for children. This is consistent with the previous works. However, it also shows that PAYG pension scheme discourages the self-education. This suggests that considering the self-education is meaningful.

## **4. Comparative Statics**

In this section, we analyze the equilibrium. At first, we investigate the relationship between the gross investments of human capital and the contribution rate.

We denote the gross investments,  $((1-u)e^\beta + ug^\beta)^{\frac{\alpha}{\beta}} = H$  .

$$\text{We have } H = \left( (1-u)^{\frac{1}{1-\beta}} (1-\theta)^{\frac{\beta}{1-\beta}} + u^{\frac{1}{1-\beta}} \theta^{\frac{\beta}{1-\beta}} \right)^{\frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha}} .$$

$$\text{Therefore, } \frac{\partial H}{\partial \theta} = \frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha} \left( (1-u)^{\frac{1}{1-\beta}} (1-\theta)^{\frac{\beta}{1-\beta}} + u^{\frac{1}{1-\beta}} \theta^{\frac{\beta}{1-\beta}} \right)^{\frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha}-1} \cdot \left( - (1-u)^{\frac{1}{1-\beta}} (1-\theta)^{\frac{2\beta-1}{1-\beta}} + u^{\frac{1}{1-\beta}} \theta^{\frac{2\beta-1}{1-\beta}} \right).$$

$$\text{We denote } \hat{\theta} = \frac{1}{1 + \left(\frac{u}{1-u}\right)^{\frac{1}{2\beta-1}}}.$$

**Lemma 1**

i) When  $\beta < 1/2$ ,  $\frac{\partial H}{\partial \theta} > 0$  when  $\theta < \hat{\theta}$  and  $\frac{\partial H}{\partial \theta} < 0$  when  $\theta > \hat{\theta}$ .

ii) When  $\beta > 1/2$ ,  $\frac{\partial H}{\partial \theta} < 0$  when  $\theta < \hat{\theta}$  and  $\frac{\partial H}{\partial \theta} > 0$  when  $\theta > \hat{\theta}$ .

Proof: See Appendix.

From Proposition 1, the self-education decreases and the education for children as the contribution rate increases. The lemma 1 suggests how the contribution rate affects the accumulation of the human capital and that  $\beta$ , which represents the elasticity of substitution, has the important role in the accumulation. When  $\beta$  is larger, the concentration for the only production factor is more efficient. In the other side, if  $\beta$  is smaller, the balance of the two factors is more efficient. From Proposition 1, the increase of the contribution rate induces the decrease of the self-education and the increase of the education for children. Therefore, the optimal contribution rate is 0 or 1 when  $\beta$  is large and the optimal contribution rate is between 0 and 1 when  $\beta$  is small. We denote  $\theta^* = \arg \max_{\theta} H$ . We have the following proposition.

**Proposition 2**

- i) When  $\beta < \frac{1}{2}$ ,  $\theta^* = \hat{\theta}$ .
- ii) When  $\beta \geq \frac{1}{2}$ ,  $\theta^* = 0$  if  $u < \frac{1}{2}$   
 $\theta^* = 1$  if  $u > \frac{1}{2}$ .

Proof: See Appendix.

When  $\beta < \frac{1}{2}$ , the gross investment of human capital increases as the contribution rate increases in  $\theta < \theta^*$ . Furthermore, in  $\theta > \theta^*$ , the gross investment decreases as the contribution rate increases. When  $\beta \geq \frac{1}{2}$ , the gross investment decreases (increases) as the contribution rate increases when  $\theta$  is small (large). So, we have the following proposition.

**Proposition 3**

*The increase of the contribution rate hampers enhancing human capital when (1)  $\beta < \frac{1}{2}$  and  $\theta > \hat{\theta}$  or (2)  $\beta > \frac{1}{2}$  and  $\theta < \hat{\theta}$  or (3)  $\beta = \frac{1}{2}$  and  $u < \frac{1}{2}$ .*

Proposition 3 means that the increase of the contribution rate can inhibit accumulating human capital. Specially, when  $\beta$  is large, the increase of the contribution rate inhibits accumulating human capital even if the contribution rate is small. The proposition implies that the scale of the PAYG pension scheme reduces economic growth when we consider the incentives to self-educate. In other words, the PAYG pension scheme can hamper economic growth.

**5. Concluding Remarks**

In this paper, we show that the PAYG pension scheme hampers accumulating human



capital when the isoquant is curved when the incentives for self-education are considered. This conclusion is opposite to those of studies such as Kaganovich and Zilcha (2008), Sánchez-Losada (2000), and Kaganovich and Zilcha (2008).

In fact, the PAYG pension scheme encourages educating children. However, the scheme discourages the incentive to self-educate by taxing human capital in the future. When we consider the incentives for self-educating, we suggest that the PAYG pension scheme leads to education-minded parents and motivationally deficient students.

Toya (1998) shows that the increase in government expenditure in primary and secondary education promotes economic growth and that the increase in government expenditure in tertiary education hampers economic growth. Saito (2005) reveals that the real effects of higher education on wages range from slightly negative to insignificant. Both of these studies presume that universities, as educational institutions, play a role of screening, rather than that of accumulating human resources, as shown by Spence (1973). Specifically, their presumption is that students at universities are competent not because they are provided better education, but because the universities themselves only select the most competent students for their courses on the basis of entrance examinations. As Gradstein, Justman, and Meier (2005) admit, the screening device helps employers identify the potential productivity of their prospective employees. The screening clearly contributes private returns while it does not directly affect social productivity. Therefore, Toya (1998) concludes that universities may not contribute to social productivity. However, in this paper, we theoretically imply that the existence of children's self-education can induce the phenomenon to seem that the productivity of tertiary education is lower than one of the primary education.

In tertiary education and on-the-job-training, the accumulation of human capital may be

affected by the volume of self-education, rather than the volume of education imparted to children. This argument implies that while studying the relationship between the pension scheme, education, and economic growth, we must focus on determining the decision maker of education and on the function of the human capital.

Finally, we must admit that our model is restrictive. This paper and many previous works assume that the pension contribution rate is decided exogenously. However, the rate is changed endogenously through political processes. The falling birth rate and the aging population may enhance the decision-making power of old aged people. Besides, the timing of change in the contribution rate does not always correspond with the timing of the decision to change the contribution rate. This inconsistency in the timing of the policy can lead to underinvestment in self-education because the old have the incentive to change the contribution rate so as to increase their pension benefit after the young make the decision regarding the volume of self-education. Furthermore, we assume that each generation includes an egoistic representative individual because we focus on intergenerational conflict, and not on intra-generational conflict. Considering the intra-generational conflict, a free-rider for not bearing children, but receiving pension from other people's children may emerge. Therefore, investment in human capital will be reduced in the model with intra-generational conflict. Needless to say, altruism can be also a solution under these conflicts. Further investigation of such endogenous political decision making, endogenous timing, intra-generational conflict, altruism, and the mutual effect among them should be conducted in future research.

## **A. Appendix**

### **A.1 Proof of Proposition 1**

$$g^{\alpha-1} = (1-\theta)^{\frac{\alpha-1}{\beta-1}} (1-u)^{\frac{\alpha-1}{\beta-1}} \cdot f(\theta)^{\alpha-1}$$

$$\begin{aligned}
&= \frac{1+r}{\alpha w} (1-\theta)^{\frac{\alpha-1}{\beta-1}} (1-u)^{\frac{\alpha-1}{\beta-1}} \frac{1}{\theta} \frac{1}{1-\theta} \frac{1}{u} \frac{1}{1-u} \cdot \\
&\quad \left\{ u(1-\theta)^{\frac{\beta}{\beta-1}} (1-u)^{\frac{\beta}{\beta-1}} \left( \frac{1-u}{u} \left( \frac{\theta}{1-\theta} \frac{u}{1-u} \right)^{\frac{\beta}{\beta-1}} + 1 \right) \right\}^{1-\frac{\alpha}{\beta}} \\
&= \frac{1+r}{\alpha w} \frac{1}{\theta} u^{-\frac{\alpha}{\beta}} \left( \frac{1-u}{u} \left( \frac{\theta}{1-\theta} \frac{u}{1-u} \right)^{\frac{\beta}{\beta-1}} + 1 \right)^{1-\frac{\alpha}{\beta}}.
\end{aligned}$$

Because  $\frac{\partial}{\partial \theta} g^{\alpha-1} = \frac{\partial}{\partial \theta} \left\{ \frac{1}{\theta} \left( \frac{1-u}{u} \left( \frac{\theta}{1-\theta} \frac{u}{1-u} \right)^{\frac{\beta}{\beta-1}} + 1 \right)^{1-\frac{\alpha}{\beta}} \right\} < 0$ ,  $\frac{\partial g}{\partial \theta} > 0$ .

$$\begin{aligned}
e^{\alpha-1} &= \theta^{\frac{\alpha-1}{\beta-1}} u^{\frac{\alpha-1}{\beta-1}} \cdot f(\theta)^{\alpha-1} \\
&= \frac{1+r}{\alpha w} \theta^{\frac{\alpha-1}{\beta-1}} u^{\frac{\alpha-1}{\beta-1}} \frac{1}{\theta} \frac{1}{1-\theta} \frac{1}{u} \frac{1}{1-u} \cdot \\
&\quad \left\{ u \theta^{\frac{\beta}{\beta-1}} u^{\frac{\beta}{\beta-1}} \left( \frac{u}{1-u} \left( \frac{1-\theta}{\theta} \frac{1-u}{u} \right)^{\frac{\beta}{\beta-1}} + 1 \right) \right\}^{1-\frac{\alpha}{\beta}} \\
&= \frac{1+r}{\alpha w} \frac{1}{1-\theta} (1-u)^{-\frac{\alpha}{\beta}} \left( \frac{u}{1-u} \left( \frac{1-\theta}{\theta} \frac{1-u}{u} \right)^{\frac{\beta}{\beta-1}} + 1 \right)^{1-\frac{\alpha}{\beta}}.
\end{aligned}$$

Because  $\frac{\partial}{\partial \theta} e^{\alpha-1} = \frac{\partial}{\partial \theta} \left\{ \frac{1}{1-\theta} \left( \frac{u}{1-u} \left( \frac{1-\theta}{\theta} \frac{1-u}{u} \right)^{\frac{\beta}{\beta-1}} + 1 \right)^{1-\frac{\alpha}{\beta}} \right\} > 0$ ,  $\frac{\partial e}{\partial \theta} < 0$ .

## A.2 Proof of Lemma 1

From  $0 < \alpha < \beta < 1$ ,  $0 < u < 1$ , and  $0 < \theta < 1$ ,

$$\frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha} \left( (1-u)^{\frac{1}{1-\beta}} (1-\theta)^{\frac{\beta}{1-\beta}} + u^{\frac{1}{1-\beta}} \theta^{\frac{\beta}{1-\beta}} \right)^{\frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha}} > 0.$$

i) When  $\beta < \frac{1}{2}$ ,

$$\text{if } \theta < \hat{\theta}, \quad -(1-u)^{\frac{1}{1-\beta}} (1-\theta)^{\frac{2\beta-1}{1-\beta}} + u^{\frac{1}{1-\beta}} \theta^{\frac{2\beta-1}{1-\beta}} > 0. \quad \therefore \frac{\partial H}{\partial \theta} > 0.$$

$$\text{if } \theta > \hat{\theta}, \quad -(1-u)^{\frac{1}{1-\beta}} (1-\theta)^{\frac{2\beta-1}{1-\beta}} + u^{\frac{1}{1-\beta}} \theta^{\frac{2\beta-1}{1-\beta}} < 0. \quad \therefore \frac{\partial H}{\partial \theta} < 0.$$

ii) When  $\beta > \frac{1}{2}$ ,

$$\text{if } \theta < \hat{\theta}, \quad -(1-u)^{\frac{1}{1-\beta}} (1-\theta)^{\frac{2\beta-1}{1-\beta}} + u^{\frac{1}{1-\beta}} \theta^{\frac{2\beta-1}{1-\beta}} < 0. \quad \therefore \frac{\partial H}{\partial \theta} < 0.$$

$$\text{if } \theta > \hat{\theta}, \quad -(1-u)^{\frac{1}{1-\beta}} (1-\theta)^{\frac{2\beta-1}{1-\beta}} + u^{\frac{1}{1-\beta}} \theta^{\frac{2\beta-1}{1-\beta}} > 0. \quad \therefore \frac{\partial H}{\partial \theta} > 0.$$

### A.3 Proof of Proposition 2

a) When  $\beta = \frac{1}{2}$ ,

$$-(1-u)^{\frac{1}{1-\beta}} (1-\theta)^{\frac{2\beta-1}{1-\beta}} + u^{\frac{1}{1-\beta}} \theta^{\frac{2\beta-1}{1-\beta}} = -(1-u)^2 (1-\theta)^0 + u^2 \theta^0 = -1 + 2u.$$

Therefore,

$$\frac{\partial H}{\partial \theta} = \begin{cases} \frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha} \left( (1-u)^{\frac{1}{1-\beta}} (1-\theta)^{\frac{\beta}{1-\beta}} + u^{\frac{1}{1-\beta}} \theta^{\frac{\beta}{1-\beta}} \right)^{\frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha} - 1} (-1+2u) < 0 & \text{if } u < \frac{1}{2} \\ \frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha} \left( (1-u)^{\frac{1}{1-\beta}} (1-\theta)^{\frac{\beta}{1-\beta}} + u^{\frac{1}{1-\beta}} \theta^{\frac{\beta}{1-\beta}} \right)^{\frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha} - 1} (-1+2u) > 0 & \text{if } u > \frac{1}{2}. \end{cases}$$

$$\therefore \theta^* = 0 \text{ if } u < \frac{1}{2}$$

$$\theta^* = 1 \text{ if } u > \frac{1}{2}.$$

b) When  $\beta < \frac{1}{2}$ ,

From Lemma 1,  $\frac{\partial H}{\partial \theta} > 0$  when  $\theta < \hat{\theta}$  and  $\frac{\partial H}{\partial \theta} < 0$  when  $\theta > \hat{\theta}$ .

Because  $0 < \hat{\theta} < 1$ ,  $\arg \max_{\theta} H = \hat{\theta}$ .  $\therefore \theta^* = \hat{\theta}$ .

c) When  $\beta > \frac{1}{2}$ ,

From Lemma 1,  $\frac{\partial H}{\partial \theta} < 0$  when  $\theta < \hat{\theta}$  and  $\frac{\partial H}{\partial \theta} > 0$  when  $\theta > \hat{\theta}$ .

Because  $0 < \hat{\theta} < 1$ ,  $\arg \max_{\theta} H = 0$  or  $1$ .

$$\text{If } \theta=0, H = \left( (1-u)^{\frac{1}{1-\beta}} \right)^{\frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha}} = (1-u)^{\frac{\alpha}{1-\alpha}}.$$

$$\text{If } \theta=1, H = \left( u^{\frac{1}{1-\beta}} \right)^{\frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha}} = u^{\frac{\alpha}{1-\alpha}}.$$

$$\begin{aligned} \therefore \theta^* &= 0 \text{ if } u < \frac{1}{2} \\ \theta^* &= 1 \text{ if } u > \frac{1}{2}. \end{aligned}$$

#### A.4 Proof of Proposition 3

When  $\beta < \frac{1}{2}$ , if  $\theta < \hat{\theta}$ ,  $\frac{\partial H}{\partial \theta} > 0$  and if  $\theta > \hat{\theta}$ ,  $\frac{\partial H}{\partial \theta} < 0$ .

Furthermore, when  $\beta > \frac{1}{2}$ , if  $\theta < \hat{\theta}$ ,  $\frac{\partial H}{\partial \theta} < 0$  and if  $\theta > \hat{\theta}$ ,  $\frac{\partial H}{\partial \theta} > 0$ .

Now,  $H$  is the monotone function of the contribution rate when  $\beta = \frac{1}{2}$ .

$u < \frac{1}{2}$  implies  $\frac{\partial H}{\partial \theta} < 0$ .

## References

- Becker, G. (1993) *Human capital: A theoretical and empirical analysis, with special reference to education* 3rd Ed.: University of Chicago Press (1st ed.:1964).
- Blanchard, O.J. (1990) "Discussion" Dornbusch, R. and Draghi M. eds., *Public debt management: theory and history*, 47-51.
- Docquier, F. and Paddison, O. (2003) "Social Security Benefit Rules, Growth and Inequality," *Journal of Macroeconomics*, 25, 47-71.
- Feldstein, M. (1974) "Social Security, Induced Retirement, and Aggregate Capital Accumulation," *The Journal of Political Economy*, 82, 5, 905-926.
- Gradstein, M., Justman, M., and Meier, V. (2005) *The Political Economy of Education*: Massachusetts Institute of Technology Press.
- Kaganovich, M. and Zilcha, I. (2008) "PAYG Pensions, Public Education and Growth" CESifo Conference Centre.
- Kemnitz, A. and Wigger, B.U. (2000) "Growth and Social Security: the Role of Human Capital," *European Journal of Political Economy*, 16, 673-683.
- Pogue, T.F. and Sgontz, L.G. (1977) "Social Security and Investment in Human Capital," *National Tax Journal*, 30, 157-169.
- Saito, K. (2005) "Do Schools Form Human Capital? Distributional Divide and

Cohort-based Analysis in Japan,” *COE Discussion Paper*, COE-F-82, Faculty of Economics, University of Tokyo.

Spence, M. (1973) “Job Market Signaling,” *The Quarterly Journal of Economics*, 87, 3, 355-374.

Toya, H. (1998) “Cross-country niokeru Zinteki Shihon to Keizaiseicho no Zisshobunseki (The Empirical Analysis of the Relationship between Human Capital and Economic Growth with Cross-Country Data)”, *Financial Review*, 46, Ministry of Finance, Japan.